

# Heating of the Solar Corona by Alfvén Waves: Self-Induced Opacity<sup>1</sup>

N. I. Zahariev and T. M. Mishonov

*Department of Theoretical Physics, Faculty of Physics,  
St. Clement of Ohrid University at Sofia,  
5 James Bourchier Blvd., BG-1164 Sofia, Bulgaria  
e-mails: nedeltcho.zahariev@gmail.com, tmishonov@phys.uni-sofia.bg*

**Abstract.** There have been derived equations describing the static distributions of temperature and wind velocity at the transition region within the framework of the magnetohydrodynamics (MHD) of fully ionized hydrogen plasma. We have also calculated the width of the transition between the chromosphere and corona as a self-induced opacity of the high-frequency Alfvén waves (AWs). The domain wall is a direct consequence of the self-consistent MHD treatment of AWs propagation. We predict considerable spectral density of the high-frequency AWs in the photosphere. The idea that Alfvén waves might heat the solar corona belong to Alfvén—we simply derived the corresponding MHD equations. The comparison of the solutions to those equations with the observational/measured data will be crucial for revealing the heating mechanism. The analysis of those solutions will explain how Alfvén waves brick unto the corona and dissipate their energy there.

**Keywords:** solar corona, Alfvénic and magnetosonic waves, coronal heating

**PACS:** 96.60.P-, 96.60.Vg, 90.60.Xy

## 1. ALFVÉN MODEL FOR CORONAL HEATING

The discovery of the lines of the multiply ionized iron in the solar corona spectrum [1] posed an important problem for the fundamental physics—what is the mechanism of heating the solar corona and why the temperature of the corona is 100 times larger than the temperature of the photosphere.

The first idea by Alfvén [2] was that Alfvén waves (AWs) [3] are the mechanism for heating the corona. AWs are generated by the turbulence in the convection zone and propagate along the magnetic field lines. Absorption is proportional to  $\omega^2$  and the heating comes from the high-frequency AWs. The low frequency AWs reach the Earth orbit and thanks to the magnetometers on the various satellites we “hear” the basses of the great symphony of the solar turbulence.

The purpose of the present work is to examine whether the initial idea is correct and to derive the MHD equations which give the dependence of the temperature on the height  $T(x)$  and the related velocity of the solar wind  $U(x)$ .

Our starting point are the MHD equations for the velocity field  $\mathbf{v}$  and magnetic field  $\mathbf{B}$

$$\partial_t \rho + \text{div} \mathbf{j} = 0, \quad \mathbf{j} = \rho \mathbf{v}, \quad (1)$$

$$\partial_t \left( \frac{1}{2} \rho \mathbf{v}^2 + \varepsilon + \frac{\mathbf{B}^2}{2\mu_0} \right) + \text{div} \mathbf{q} = 0, \quad (2)$$

$$\partial_t (\rho \mathbf{v}) + \nabla \cdot \Pi = 0, \quad (3)$$

where

$$\mathbf{q} = \rho \left( \frac{1}{2} \mathbf{v}^2 + h \right) \mathbf{v} + \mathbf{v} \cdot \Pi^{\text{visc}} - \kappa \nabla T + \mathbf{S} \quad (4)$$

is the energy density flux,  $\rho$  is the mass density,  $\varepsilon$  is the internal energy density,  $\kappa$  is the thermal conductivity,  $h$  is the enthalpy per unit mass;

$$\mathbf{S} = \frac{1}{\mu_0} [\mathbf{B} \times (\mathbf{v} \times \mathbf{B}) - v_m \mathbf{B} \times (\nabla \times \mathbf{B})], \quad (5)$$

<sup>1</sup> This paper should be cited as published in the Proceedings of the School and Workshop on Space Plasma Physics (1–12 September 2010, Kiten, Bulgaria), AIP Conference Proceedings 1356, American Institute of Physics, Melville, NY, 2011.

is the Poynting vector and  $v_m \equiv c^2 \epsilon_0 \rho_\Omega$  is the magnetic diffusion determined by the Ohmic resistance  $\rho_\Omega$  and vacuum susceptibility  $\epsilon_0$ ; vacuum permeability is  $\mu_0$ . For hot enough plasma  $\rho_\Omega$  is negligible and we ignore it hereafter. The total momentum flux

$$\mathbf{\Pi} = \rho \mathbf{v} \mathbf{v} + \mathbf{P} + \mathbf{\Pi}^{\text{visc}} + \mathbf{\Pi}^{\text{Maxw}} \quad (6)$$

is a sum of the inviscid part  $\rho \mathbf{v} \mathbf{v} + \mathbf{P}$  of the fluid, with pressure tensor  $\mathbf{P} = p \mathbf{I}$ ,

$$\Pi_{ik}^{\text{visc}} = -\eta \left( \partial_i v_k + \partial_k v_i - \frac{2}{3} \delta_{ik} \nabla \cdot \mathbf{v} \right) - \zeta \delta_{ik} \nabla \cdot \mathbf{v}, \quad (7)$$

the viscous part of the stress tensor, with viscosity  $\eta$  and second viscosity  $\zeta$ , and lastly, the Maxwell stress tensor

$$-\Pi_{ik}^{\text{Maxw}} = \frac{1}{\mu_0} \left( B_i B_k - \frac{1}{2} \mathbf{B}^2 \delta_{ik} \right), \quad (8)$$

with  $\delta_{ik}$  the Kronecker delta. We model coronal plasma with completely ionized hydrogen plasma

$$\begin{aligned} \kappa &= 0.9 \frac{T^{5/2}}{e^4 m_e^{1/2} \Lambda}, \quad \eta = 0.4 \frac{m_p^{1/2} T^{5/2}}{e^4 \Lambda}, \quad \zeta \approx 0, \\ \Lambda &= \ln \left( \frac{r_D T}{e^2} \right), \quad \frac{1}{r_D^2} = \frac{4\pi e^2 n_{\text{tot}}}{T}, \quad e^2 \equiv \frac{q_e^2}{4\pi \epsilon_0}, \end{aligned} \quad (9)$$

where  $q_e$  is the electron charge,  $m_e$  is the mass of electron,  $m_p$  is the proton mass,  $T$  is the temperature and  $n_{\text{tot}} = n_e + n_p$  is the total number density of electrons and protons;  $\rho = m_p n_p$ . We suppose that  $\mu_0 = 4\pi$  and  $\epsilon_0 = 1/4\pi$ , but in the practical system all formulas are the same; as well as in Heaviside–Lorentz units where  $\mu_0 = 1$  and  $\epsilon_0 = 1$ . As we mentioned above

$$v_m = \frac{c^2}{4\pi} \frac{e^2 m_e^{1/2} \Lambda}{0.6 T^{3/2}} \ll v_k \equiv \frac{\eta}{\rho} = \frac{0.4 T^{5/2}}{e^4 m_p^{1/2} n_p \Lambda}; \quad (10)$$

i.e., the hot hydrogen plasma is sticky, dilute, and “superconducting,”  $v_m \approx 0$ . Let us mention also the relations  $\kappa \rho_\Omega = 1.5 T / q_e^2$  and  $\eta / \kappa \approx \frac{4}{9} \sqrt{m_e m_p}$ ,

$$\rho_\Omega = \frac{1}{4\pi \epsilon_0} \frac{e^2 m_e^{1/2} \Lambda}{0.6 T^{3/2}}. \quad (11)$$

## 2. MAGNETOHYDRODYNAMIC EQUATIONS AND ENERGY FLUXES

The time derivative  $\partial_t \mathbf{B}$  which implicitly participate in the energy conservation Eq. (2) at zero Ohmic resistivity obeys the equation

$$\mathbf{d}_t \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v}, \quad \mathbf{d}_t \equiv \partial_t + \mathbf{v} \cdot \nabla. \quad (12)$$

Analogously the momentum equation (3) can be rewritten by the substantial derivative

$$\rho \mathbf{d}_t v_i = -\partial_i p + \partial_k \left\{ \eta \left( \partial_k v_i + \partial_i v_k - \frac{2}{3} \delta_{ik} \partial_j v_j \right) \right\} + \partial_i (\zeta \partial_j v_j) - \frac{1}{\mu_0} (\mathbf{B} \times \text{rot} \mathbf{B})_i. \quad (13)$$

In our model we consider AWs propagating along magnetic field lines  $\mathbf{B}_0$ . We focus our attention on the narrow transition layer, where the static magnetic field is almost homogeneous and the waves are within acceptable accuracy one-dimensional. For the velocity and magnetic fields we assume

$$\begin{aligned} \mathbf{v}(t, x) &= U(x) \hat{\mathbf{x}} + u(t, x) \hat{\mathbf{z}}, \\ \mathbf{B}(t, x) &= B_0 \hat{\mathbf{x}} + b(t, x) \hat{\mathbf{z}}, \end{aligned} \quad (14)$$

with homogeneous magnetic field  $B_0 \hat{\mathbf{x}}$  perpendicular to the surface of the Sun. The transverse wave amplitudes of the velocity  $u(t, x)$  and magnetic field  $b(t, x)$  we represent with the Fourier integrals

$$u(t, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{u}(\omega, x) e^{-i\omega t} d\omega, \quad (15)$$

$$b(t, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{b}(\omega, x) e^{-i\omega t} d\omega. \quad (16)$$

For illustrative purposes it is convenient to consider monochromatic AWs with  $u(t, x) = \hat{u}(x)e^{-i\omega t}$  and  $b(t, x) = \hat{b}(x)e^{-i\omega t}$ .

## 2.1. Wave equations

For linearized waves the general MHD equations (13) and (12) give the following system for  $\hat{u}(x)$  and  $\hat{b}(x) \equiv \hat{b}(x)/B_0$

$$(-i\omega + U d_x) \hat{u} = V_A^2 d_x \hat{b} + \frac{1}{\rho} d_x (\eta d_x \hat{u}), \quad (17)$$

$$-i\omega \hat{b} = d_x \hat{u} - d_x (U \hat{b}), \quad (18)$$

where

$$V_A(x) = B_0 / \sqrt{\mu_0 \rho(x)} \quad (19)$$

is the Alfvén velocity. In our numerical analysis we solve the first order linear set of equations

$$\begin{aligned} -i d_x |\Psi\rangle &= \frac{i}{v_k U} M |\Psi\rangle = K |\Psi\rangle, \\ |\Psi\rangle &\equiv \begin{pmatrix} \hat{u} \\ \hat{b} \\ \hat{w} \end{pmatrix}, \quad K = \frac{i}{v_k U} M, \\ \langle \Psi | &\equiv |\Psi\rangle^\dagger = (\hat{u}^*, \hat{b}^*, \hat{w}^*), \end{aligned} \quad (20)$$

where  $\hat{w} \equiv d_x \hat{u}$ , and

$$M \equiv \begin{pmatrix} 0 & 0 & -v_k U \\ 0 & v_k(-i\omega + d_x U) & -v_k \\ i\omega U & -V_A^2(-i\omega + d_x U) & (V_A^2 - U^2) + \frac{U}{\rho} d_x \eta \end{pmatrix}.$$

For homogeneous medium with constant  $\eta$ ,  $\rho$ ,  $V_A$ , and  $U$ , in short for constant wavevector matrix  $K$ , the exponential substitution  $\Psi \propto \exp(ikx)$  in Eq. (20) or equivalently Eqs. (17) and (18) gives the secular equation

$$i v_k U \det(K - kl) = \omega_D (\omega_D + i v_k k^2) - V_A^2 k^2 = 0, \quad (21)$$

where  $\omega_D \equiv \omega - kU$  is the Doppler shifted wave frequency. This secular equation gives the well-known dispersion  $\omega_D (\omega_D + i v_k k^2) = V_A^2 k^2$  of the AWs. This equation is quadratic with respect to  $\omega$  and cubic with respect to  $k$ .

## 2.2. Wind variables

We solve the wave equation (20) from “Sun” surface  $x = 0$  to some distance large enough  $x = l$ , where the short-wavelength AWs are almost absorbed. This distance is much bigger than the *width of the transition layer*  $\lambda$ , but much smaller than the solar radius. The considered one-dimensional  $0 < x < l$  time-independent problem has three integrals corresponding to the three conservation laws related to mass, energy and momentum. The mass conservation law Eq. (1) gives the constant flow

$$j = \rho(x)U(x) = \rho_0 U_0 = \rho_l U_l = \text{const}, \quad (22)$$

where  $\rho_0 = \rho(0)$ ,  $\rho_l = \rho(l)$ ,  $U_0 = U(0)$ , and  $U_l = U(l)$ . The energy conservation law reduces to a constant flux along the  $x$ -axis

$$q = q_{\text{wind}}^{\text{ideal}}(x) + \tilde{q}(x) = \rho U \left( \frac{1}{2} U^2 + h \right) + \tilde{q} = \text{const}. \quad (23)$$

Here the first term describes the energy of the ideal wind, i.e., a wind from an ideal (inviscid) fluid. The second term  $\tilde{q}(x)$  includes all other energy fluxes; in our notations tilde will denote sum of the non-ideal (dissipative) terms of the

wind and wave terms. In detail the non-ideal part of the energy flux  $\tilde{q}(x)$  consists of: the wave kinetic energy  $\propto |\hat{u}|^2$ , viscosity (wind  $\propto \frac{4}{3}\eta + \zeta$  and wave  $\propto \eta$  components), heat conductivity  $\propto \kappa$ , and Poynting vector  $\propto \hat{b}^*$ ,

$$\tilde{q}(x) \equiv \frac{j}{4} |\hat{u}|^2 - \xi U d_x U - \frac{1}{4} \eta d_x |\hat{u}|^2 - \kappa d_x T + \frac{1}{2\mu_0} \left( U |\hat{b}|^2 - B_0 \text{Re}(\hat{b}^* \hat{u}) \right), \quad (24)$$

where  $\xi \equiv \frac{4}{3}\eta + \zeta$ . Here time averaged energy flux is represented by the amplitudes of the monochromatic oscillations, this is a standard procedure for alternating current processes. In our case we have, for example,  $\langle \hat{u}^2 \rangle_t = \langle (\text{Re} \hat{u})^2 \rangle_t = \langle \frac{1}{4} (\hat{u} + \hat{u}^*)^2 \rangle_t = \frac{1}{2} |\hat{u}|^2$ . The other terms from Eq. (4) are averaged in a similar way in the equation above.

The momentum conservation law Eq. (6) gives constant flux  $\Pi = \Pi_{xx}$

$$\Pi = \Pi_{\text{wind}}^{\text{ideal}}(x) + \tilde{\Pi}(x) = \rho U U + P + \tilde{\Pi}, \quad (25)$$

the sum of the ideal wind fluid and the other terms

$$\tilde{\Pi}(x) \equiv \frac{1}{4\mu_0} |\hat{b}|^2 - \xi d_x U, \quad (26)$$

which take into account the wave part of the Maxwell stress tensor  $\propto b^2$  and viscosity of the wind  $\propto \xi$ .

We have to solve the hydrodynamic problem for calculation of wind velocity and temperature at known energy and momentum fluxes. The problem is formally reduced to analogous one for a jet engine, cf. Ref. [4]. We approximate the corona as fully ionized hydrogen plasma, i.e., electrically neutral mixture of electrons and protons. The experimental data tell us that proton temperature  $T_p$  is higher than electron one  $T_e$ . This is an important hint that heating goes through the viscosity determined mainly by protons. However, for illustration purpose and simplicity we assume proton and electron temperatures to be equal  $T_e = T_p = T$ . For such an ideal (in thermodynamic sense) gas the local sound velocity is

$$\begin{aligned} c_s^2(x) &= \frac{c_p}{c_v} \frac{p}{\rho} = \gamma \frac{T}{\langle m \rangle}, \quad \gamma = \frac{c_p}{c_v} = \frac{5}{3}, \\ \langle m \rangle &= \frac{n_p m_p + n_e m_e}{n_p + n_e} \approx \frac{1}{2} m_p, \quad n_e = n_p = \frac{1}{2} n_{\text{tot}}, \\ p &= n_{\text{tot}} T = \frac{\rho T}{\langle m \rangle} = \frac{j}{U} \frac{T}{\langle m \rangle}, \quad h = c_p \frac{T}{\langle m \rangle} = \frac{\varepsilon + p}{\rho}, \end{aligned} \quad (27)$$

where, as we mentioned earlier,  $h$  is the enthalpy per unit mass and  $\varepsilon$  is the density of the internal energy.

In order to alleviate the final formulas we introduce two dimensionless variables  $\sigma$  and  $\tau$  which represent the non-ideal part of the energy and momentum flux respectively

$$\sigma^2(x) \equiv \frac{\tilde{q}(x)}{\rho_0 U_0^3} \Big|_x^0, \quad \tau(x) \equiv \frac{\tilde{\Pi}(x)}{\rho_0 U_0^2} \Big|_x^0, \quad (28)$$

and analogously for the wind velocity and temperature

$$\overline{U}(x) \equiv \frac{U(x)}{U_0}, \quad \Theta(x) \equiv \frac{T(x)}{\langle m \rangle U_0^2}, \quad \Theta_0 = \Theta(0), \quad (29)$$

where  $U_0 = U(0)$ . The energy and momentum constant fluxes Eq. (23) and Eq. (25) in the new notation take the form

$$\frac{q - \tilde{q}(0)}{\rho_0 U_0^3} = \frac{1}{2} \overline{U}^2 + c_p \Theta - \sigma^2 = \frac{1}{2} + c_p \Theta_0, \quad (30)$$

$$\frac{\Pi - \tilde{\Pi}(0)}{\rho_0 U_0^2} = \overline{U} + \Theta / \overline{U} - \tau = 1 + \Theta_0. \quad (31)$$

From the second equation we express the dimensionless temperature  $\Theta$  and substitute in the first one. Solving the quadratic equation for the wind velocity  $U$  we derive

$$\overline{U}(x) = \frac{1}{\gamma + 1} \left( \gamma + s^2 + \gamma \tau(x) - \sqrt{\mathcal{D}(x)} \right), \quad (32)$$

where for the discriminant we have

$$\begin{aligned}\mathcal{D} &= (s^2 - 1)^2 - 2(\gamma^2 - 1)\sigma^2 + \gamma\tau[\gamma\tau + 2(1 + s^2)], \\ s^2 &\equiv \frac{c_s^2(0)}{U_0^2} = \gamma\Theta_0, \quad c_s^2(x) = \left(\frac{\partial p}{\partial \rho}\right)_s = \frac{\gamma T(x)}{\langle m \rangle}.\end{aligned}\quad (33)$$

Here  $\gamma$  is the constant ratio of the heat capacities, and  $s \equiv c_s(0)/U_0$  is the ratio of the sound and wind velocity at  $x = 0$ . We suppose that initial wind velocity is very small  $U(0) \ll c_s(0)$ . The velocity distribution Eq. (32) can be substituted in Eq. (31) and we derive a dimensionless equation for the temperature distribution

$$\Theta(x) = \bar{U}(x) \left( 1 + \frac{s^2}{\gamma} + \gamma\tau(x) - \bar{U}(x) \right), \quad (34)$$

The solutions for velocity  $U(x)$  Eq. (32) and temperature  $T(x)$  Eq. (34) distributions are important ingredients in our analysis and derivation of a selfconsistent picture of the solar wind.

### 2.3. Entropy generation and equation for the temperature distribution

Dissipated power density is equal to  $\rho T$  times the substantial derivative of entropy per unit mass

$$\rho T d_t \mathcal{S} = \text{div}(\varkappa \nabla T) + \rho_\Omega \left( \frac{\text{rot} \mathbf{B}}{\mu_0} \right)^2 + \sigma'_{ik} \partial_k v_i. \quad (35)$$

From the right-hand side of this equation for the heat transfer the first term describes the thermal conductivity, the second one is the Ohmic power which is negligible  $\rho_\Omega \approx 0$ , and the last term gives the heating by viscous part of the strength tensor

$$\sigma'_{ik} = \eta \left( \partial_i v_k + \partial_k v_i - \frac{2}{3} \delta_{ik} \text{div} \mathbf{v} \right) + \zeta \delta_{ik} \text{div} \mathbf{v}. \quad (36)$$

The time averaged density of viscous heating power is a sum of wind and wave parts

$$\langle \sigma'_{ik} \partial_k v_i \rangle_t = \xi (d_x U)^2 + \frac{1}{4} \eta |\hat{w}|^2 = \frac{\rho_0 U_0^3}{l} \left( \bar{\xi} \bar{W}^2 + \frac{1}{4} \bar{\eta} |\hat{w}|^2 \right), \quad (37)$$

where

$$\bar{\xi} = \frac{4}{3} \bar{\eta} + \bar{\zeta} \quad \text{with} \quad \rho_\Omega = 0.$$

We solve the transition layer problem in a space interval  $0 < x < l$  and introduce convenient for further work dimensionless variables

$$\begin{aligned}\bar{x} &= \frac{x}{l}, \quad \bar{W} = d_{\bar{x}} \bar{U} = \frac{d_x U}{l U_0}, \quad \hat{w} = d_{\bar{x}} \hat{u} = \frac{d_x \hat{u}}{l U_0}, \\ \bar{\eta} &= \frac{\eta}{\rho_0 l U_0}, \quad \bar{\zeta} = \frac{\zeta}{\rho_0 l U_0}, \quad \bar{\varkappa} = \frac{\langle m \rangle U_0^2 \varkappa}{l \rho_0 U_0^3}.\end{aligned}\quad (38)$$

Analogously introducing reduced temperature and dimensionless temperature gradient  $\bar{F}$

$$\bar{T} \equiv \frac{T}{T_0} = \frac{\Theta}{\Theta_0}, \quad F = d_x T, \quad \bar{F} = d_{\bar{x}} \bar{T}. \quad (39)$$

we rewrite the heating power density by heat conductivity as

$$\text{div}(\varkappa \nabla T) = \frac{\rho_0 U_0^3 \Theta_0}{l} \left[ (d_{\bar{x}} \bar{\varkappa}) \bar{F} + \bar{\varkappa} d_{\bar{x}} \bar{F} \right]. \quad (40)$$

The hot corona we model as ideal gas

$$S = N \ln \frac{V}{N} + N c_V \ln T + N(c_V + \text{const}), \quad (41)$$

of electrons and protons

$$\mathcal{S} = \frac{S_e + S_p}{m_p N_p + m_e N_e}, \quad N_p = n_p V, \quad N_e = n_e V.$$

The substitution here  $n_e = n_p = \frac{1}{2} n_{\text{tot}}$  gives for the time derivative

$$\langle d_t \mathcal{S}(x) \rangle_t = \mathbf{v} \cdot \nabla \mathcal{S} = \frac{U}{\langle m \rangle} \left( -\frac{d_x n_{\text{tot}}}{n_{\text{tot}}} + c_V \frac{d_x T}{T} \right), \quad (42)$$

where time averaging is over wave period. In such a way taking into account the mass conservation law  $j = \langle m \rangle n_{\text{tot}} U$  we obtain for the irreversible heating power

$$\begin{aligned} \rho T d_t \mathcal{S} &= \frac{T}{\langle m \rangle} (\rho U = j) \left[ \frac{d_x U}{U} + \left( c_V = \frac{3}{2} \right) \frac{d_x T}{T} \right] \\ &= \frac{\rho_0 U_0^3 \Theta_0}{l} \left( \frac{\bar{W}}{\bar{U}} + \frac{c_V \bar{F}}{\bar{F}} \right) \bar{T}. \end{aligned} \quad (43)$$

The substitution of this equation together with Eq. (40) and Eq. (37) in Eq. (35) gives an explicit expression dimensionless temperature gradient

$$\bar{\mathcal{K}} d_{\bar{x}} \bar{F} = (c_V - d_{\bar{x}} \bar{\mathcal{K}}) \bar{F} + \frac{\bar{W} \bar{T}}{\bar{U}} - \frac{\bar{\xi}}{\Theta_0} \bar{W}^2 - \frac{\bar{\eta}}{4 \Theta_0} |\hat{w}|^2. \quad (44)$$

The thermal conductivity Eq. (9) using the dimensionless variables can be rewritten as

$$\bar{\eta} = \bar{\eta}_0 \frac{\Lambda_0}{\Lambda} \bar{T}^{5/2}, \quad \bar{\eta}_0 = \frac{\eta_0}{\rho_0 l U_0}, \quad (45)$$

$$\bar{\mathcal{K}} = \bar{\mathcal{K}}_0 \frac{\Lambda_0}{\Lambda} \bar{T}^{5/2}, \quad \bar{\mathcal{K}}_0 = \frac{\langle m \rangle \kappa_0}{\rho_0 l U_0}, \quad (46)$$

$$\Lambda = \Lambda_0 + \frac{3}{2} \ln \bar{T} + \frac{1}{2} \ln \bar{U}, \quad \Lambda_0 = \Lambda(x=0).$$

A simple differentiation gives

$$d_{\bar{x}} \Lambda = \frac{3}{2} \frac{\bar{F}}{\bar{T}} + \frac{1}{2} \frac{\bar{W}}{\bar{U}}, \quad (47)$$

$$\frac{d_x \bar{\mathcal{K}}}{\bar{\mathcal{K}}} = \frac{d_{\bar{x}} \bar{\eta}}{\bar{\eta}} = \left( \frac{5}{2} - \frac{3}{2 \mathcal{L}} \right) \frac{\bar{F}}{\bar{T}} - \frac{1}{2 \mathcal{L}} \frac{\bar{W}}{\bar{U}}. \quad (48)$$

Then the gradient of the kinetic coefficient in Eq. (44) can be expressed by its logarithmic derivative and we have not differentiations in the equation which have to be solved numerically. For the wind velocity we can proceed in a similar manner.

## 2.4. Momentum equation for the wind velocity

The x-component of momentum equations (3) and (13) can be also rewritten as

$$\rho (\partial_t U + U d_x U) = -d_x \left[ p + \left( \langle p_B \rangle_t = \frac{1}{4 \mu_0} |\hat{b}|^2 \right) \right] + d_x (\xi d_x U), \quad (49)$$

where we have gradient from the sum of pressure and time averaged magnetic pressure of the propagating AWs. Now we have to express all variables by dimensionless ones

$$p = n_{\text{tot}} T = \rho_0 U_0^2 \Theta_0 \frac{\bar{T}}{\bar{U}}, \quad U = U_0 \bar{U}, \quad \hat{b} = B_0 \hat{\bar{b}},$$

$$x = l \bar{x}, \quad \xi = \rho_0 l U_0 \bar{\xi}, \quad V_A = U_0 \bar{V}_A, \quad \bar{\rho} = \frac{\rho}{\rho_0} = \frac{1}{\bar{U}},$$

where

$$V_A = \frac{B_0}{\sqrt{\mu_0 \rho}} = a U_0 \sqrt{\bar{U}}, \quad a \equiv \frac{V_A(0)}{U_0} = \bar{V}_A(0) \quad (50)$$

is the Alfvén velocity. Using the relation which can be easily checked

$$a^2 = \frac{B_0^2 / \mu_0}{\rho_0 U_0^2} = \frac{\bar{V}_A^2(\bar{x})}{\bar{U}(\bar{x})} = \frac{\bar{V}_A^2(0)}{\bar{U}(0)}, \quad (51)$$

the momentum equation takes the form

$$\bar{\xi} d_{\bar{x}} \bar{W} = \left(1 - d_{\bar{x}} \bar{\xi}\right) \bar{W} - d_{\bar{x}} \left( \frac{\Theta_0 \bar{T}}{\bar{U}} + \frac{a^2}{4} |\hat{\bar{b}}|^2 \right). \quad (52)$$

Then from the second row of Eq. (20) we have

$$U d_x \hat{b} = (i\omega - W) \hat{b} + \hat{w}, \quad W = d_x U, \quad (53)$$

and easily derive

$$\bar{U} d_{\bar{x}} |\hat{\bar{b}}|^2 = -2\bar{W} |\hat{\bar{b}}|^2 + \hat{\bar{b}}^* \hat{\bar{w}} + \hat{\bar{b}} \hat{\bar{w}}^*. \quad (54)$$

Now we can substitute  $d_{\bar{x}} |\hat{\bar{b}}|^2$  in the momentum equation (52) which finally takes the derivatives free form

$$\bar{\xi} d_{\bar{x}} \bar{W} = (1 - d_{\bar{x}} \bar{\eta}) \bar{W} + \Theta_0 \left( \frac{\bar{T} \bar{W}}{\bar{U}^2} - \frac{\bar{F}}{\bar{U}} \right) + \frac{a^2}{4\bar{U}} \left( 2\bar{W} |\hat{\bar{b}}|^2 - \hat{\bar{b}}^* \hat{\bar{w}} - \hat{\bar{b}} \hat{\bar{w}}^* \right), \quad (55)$$

where we taken into account that  $\bar{U}(0) = 1$ . Let us mention also the relation between plasma  $\beta$ -parameter and dimensionless variables

$$\beta = \frac{p}{B_0^2 / 2\mu_0} = \beta_0 \frac{\bar{T}}{\bar{U}}, \quad \beta_0 = \frac{2s^2}{\gamma a^2} = \frac{2c_s^2(0)}{\gamma V_A^2(0)} = \frac{2\Theta_0}{a^2}, \quad B_0 = \sqrt{2\mu_0 n_{\text{tot}}(0) T_0 / \beta_0}. \quad (56)$$

In the model of fully ionized plasma the kinetic coefficients are also in a simple relation [5]

$$\frac{\bar{\kappa}}{\bar{\eta}} = \frac{\langle m \rangle \varkappa}{\eta} \approx \frac{0.9}{0.8} \sqrt{\frac{m_p}{m_e}} \approx 48.2 \gg 1; \quad (57)$$

the heat conductivity is determined by the fast thermal electrons while the viscosity by slowly moving protons. The ratio of kinetic coefficients is determined by the ratio of Maxwell velocities.

By definition we have the boundary condition  $\bar{U}(0) = 1$  and for qualitative modeling we can suppose small derivatives of the wind velocity in the chromosphere region  $\bar{W}(0) = 0$ . Analogously, we can suppose small gradient of the temperature  $\bar{F}(0) = 0$ . Together with  $\Theta(0) = \Theta_0 = s^2 / \gamma \gg 1$  and  $\bar{T}(0) = 1$  we have the full set of boundary conditions at  $\bar{x} = 0$  for the differential equations Eq. (55) and Eq. (44) for the wind variables  $\bar{U}$  and  $\Theta$ . More complicated are the boundary conditions for the wave variables described in the next subsection.

## 2.5. Boundary conditions for the waves

With known background wind variables  $U(x)$  and  $T(x)$  we can solve the wave equation (20) for runaway AWs at  $x = l$ . As we will see later, the runaway boundary condition Eq. (72) corresponds to right propagating AWs at the right boundary of the interval. The wave equation (20) is extremely stiff at small viscosity, and a numerical solution is possible to be obtained only downstream from  $x = 0$  to  $x = l$ . We have to find the linear combination of left and right propagating waves at  $x = 0$ , which gives the runaway condition at  $x = l$ .

The solution of wave equation according to Eq. (23) determines the energy flux related to the propagation of AWs

$$\begin{aligned} \tilde{q}_{\text{wave}}(\Psi(x)) &\equiv \langle \Psi | g | \Psi \rangle = \frac{j}{4} |\hat{u}|^2 - \frac{1}{2} \eta \text{Re}(\hat{u}^* \hat{w}) \\ &+ \frac{B_0^2}{2\mu_0} \left( U |\hat{b}|^2 - \text{Re}(\hat{b}^* \hat{u}) \right), \end{aligned} \quad (58)$$

where

$$g(x) \equiv \begin{pmatrix} \frac{1}{4}j & -\frac{B_0^2}{4\mu_0} & -\frac{1}{4}\eta \\ -\frac{B_0^2}{4\mu_0} & \frac{UB_0^2}{2\mu_0} & 0 \\ -\frac{1}{4}\eta & 0 & 0 \end{pmatrix}. \quad (59)$$

Here the  $j$ -term represent the kinetic energy of the wave,  $\eta$ -terms comes from the viscous part of the wave energy flux, and the  $B_0$ -terms describe the Pointing vector of the wave.

In order to take into account the boundary condition at  $x = l$  we calculate the eigenvectors of the matrix  $K$ , which according to Eq. (20) determine the wave propagation in a homogeneous fluid with amplitude  $\propto \exp(ikx)$ . Then the eigenvalues of  $K$  give the complex wavevectors

$$k = k' + ik'' = \text{eigenvalue}(K), \quad (60)$$

i.e.

$$\det(K - kl) = 0. \quad (61)$$

The three eigenvectors  $|L\rangle$ ,  $|D\rangle$  and  $|R\rangle$  are ordered by the spatial decrements of their eigenvalues

$$k_L'' < 0 < k_R'' < k_D'', \quad (62)$$

and are normalized by the conditions

$$-\langle L | g | L \rangle = \langle R | g | R \rangle = \langle D | g | D \rangle = 1, \quad (63)$$

where the sign corresponds to the direction of wave propagation. Notation  $L$  corresponds to left propagating wave,  $R$  to right propagating wave, and  $D$  for a overdamped at small viscosity mode.

For technical purposes we introduce the matrix notations

$$|L\rangle = \begin{pmatrix} L_u(x) \\ L_b(x) \\ L_w(x) \end{pmatrix}, \quad |R\rangle = \begin{pmatrix} R_u(x) \\ R_b(x) \\ R_w(x) \end{pmatrix}, \quad |D\rangle = \begin{pmatrix} D_u(x) \\ D_b(x) \\ D_w(x) \end{pmatrix}. \quad (64)$$

For low enough frequencies  $\omega \rightarrow 0$  and wind velocities the modes describes: 1) right-propagating AWs with  $k_R' \approx \omega/V_A$  and small  $k_R'' \approx v_k \omega^2 / 2V_A^3 \ll k_R'$ , 2) left propagating wave  $k_L = -k_R$ , and a diffusion overdamped mode  $k_D'' \approx V_A^2 / v_k U \gg k_D'$  which describes the drag of a static perturbation by the slow wind  $U \ll V_A$  in a fluid with small viscosity. In this low-frequency and long-wavelength limit the stiffness ratio of the eigenvalues is very large

$$r_{DR} = \frac{|k_D|}{|k_R|} \approx \frac{k_D''}{k_R'} \approx \frac{V_A^3}{v_k U \omega} \gg 1. \quad (65)$$

The strong inequality is applicable to the chromosphere where the viscosity of the cold plasma is very low. As we emphasized, the wave equation (20) is a very stiff system and indispensably has to be solved downstream from the chromosphere  $x = 0$  to the corona  $x = l$  using algorithms for stiff systems. Let

$$|\Psi_L(x)\rangle = \begin{pmatrix} u_L(x) \\ b_L(x) \\ w_L(x) \end{pmatrix}, \quad |\Psi_R(x)\rangle = \begin{pmatrix} u_R(x) \\ b_R(x) \\ w_R(x) \end{pmatrix} \quad (66)$$



are the solutions to the wave equation (20) with boundary conditions

$$|\psi_L(0)\rangle = |L(0)\rangle, \quad |\psi_R(0)\rangle = |R(0)\rangle. \quad (67)$$

We look for a solution as a linear combination

$$\psi(x) = \psi_R(x) + r\psi_L(x), \quad (68)$$

in other words we suppose that from the low-viscosity chromosphere plasma do not come overdamped diffusion modes. The strong decay rate make them negligible at  $x = 0$ . Physically this means that from the Sun are coming AWs (R-modes) and some of them are reflected from the transition layer (L-modes)

$$\psi(0) = |R(0)\rangle + r|L(0)\rangle. \quad (69)$$

Analogously, for the configuration of open corona we have to take into account the runaway boundary condition for which we suppose zero amplitude for the wave coming from infinity

$$\psi(l) = \tilde{t}|R(l)\rangle + \tilde{c}|D(l)\rangle. \quad (70)$$

Written by components

$$\begin{pmatrix} u_R(l) \\ b_R(l) \\ w_R(l) \end{pmatrix} + r \begin{pmatrix} u_L(l) \\ b_L(l) \\ w_L(l) \end{pmatrix} = \tilde{t} \begin{pmatrix} R_u(l) \\ R_b(l) \\ R_w(l) \end{pmatrix} + \tilde{c} \begin{pmatrix} D_u(l) \\ D_b(l) \\ D_w(l) \end{pmatrix}. \quad (71)$$

this boundary condition gives a linear system of equation for the reflection coefficient  $r$ , transmission coefficient  $\tilde{t}$  and the mode-conversion coefficient  $\tilde{c}$ .

For  $l \rightarrow \infty$  when  $\exp[-k_D''(l)l] \ll 1 \exp[-k_R''(l)l]$  the amplitude of D-mode is negligible and the runaway boundary condition reads

$$\psi(l) = \psi_R(l) + r\psi_L(l) \approx \tilde{t}|R(l)\rangle, \quad (72)$$

or by components

$$\begin{pmatrix} u_R(l) \\ \frac{u_R(l)}{b_R(l)} \end{pmatrix} + r \begin{pmatrix} u_L(l) \\ \frac{u_L(l)}{b_L(l)} \end{pmatrix} = \tilde{t} \begin{pmatrix} R_u(l) \\ R_b(l) \end{pmatrix}. \quad (73)$$

These systems give the amplitudes of the reflected wave  $r$  and transmitted wave  $\tilde{t}$  in the solution Eq. (68). For this solution we have the energy fluxes

$$\begin{aligned} \mathcal{T} &\equiv \langle \psi(l) | g(l) | \psi(l) \rangle = \\ &= |\tilde{t}|^2 + |\tilde{c}|^2 + (\tilde{t}\tilde{c}^* \langle D(l) | g(l) | R(l) \rangle + \text{c.c.}), \end{aligned} \quad (74)$$

$$\begin{aligned} 1 - \mathcal{R} &\equiv \langle \psi(0) | g(0) | \psi(0) \rangle = \\ &= 1 - |r|^2 + (r^* \langle L(0) | g(0) | R(0) \rangle + \text{c.c.}). \end{aligned} \quad (75)$$

Then we introduce the absorption coefficient

$$\mathcal{A} \equiv - \langle \psi(x) | g(x) | \psi(x) \rangle_0^l = 1 - \mathcal{R} - \mathcal{T}. \quad (76)$$

The described solution is normalized by unite energy flux of the R-wave. If we wish to fix energy flux of the right propagating wave to be  $q_{\text{wave}}(0)$  we have to make the renormalization

$$\Psi(x) = A_{\text{wave}} \psi(x). \quad (77)$$

Now using  $\Psi(x)$  we can calculate the wave part of the energy flux Eq. (58) and the wave part of the momentum flux

$$\tilde{\Pi}_{\text{wave}}(x) \equiv \frac{1}{4\mu_0} |\hat{b}(x)|^2. \quad (78)$$

This section is written in dimensional variables, but all equations can be easily converted in dimensionless variables as is done in the next sub-sub-section.

### 2.5.1. Dimensionless wave variables

Using mechanical units for length  $l$ , velocity  $U_0$  and density  $\rho_0$  we can convert all equations in dimensionless form. The formulas remain almost the same and we wish to mention only the differences. Introducing dimensionless density

$$\bar{\rho}(x) = \rho(x)/\rho_0 = 1/\bar{U}(\bar{x}) \quad (79)$$

and wave energy flux

$$\mathcal{Q}_{\text{wave}}(0) = \frac{q_{\text{wave}}(0)}{\rho_0 U_0^3} = (1 - \mathcal{R}) |A_{\text{wave}}|^2, \quad (80)$$

we have dimensionless matrices

$$\bar{\mathbf{M}} = \begin{pmatrix} 0 & 0 & -\bar{v}\bar{U} \\ 0 & \bar{v}(-i\bar{\omega} + \bar{W}) & -\bar{v} \\ i\bar{\omega}\bar{U} & -\bar{V}_A^2(-i\bar{\omega} + \bar{W}) & (\bar{V}_A^2 - \bar{U}^2) + \bar{U}^2 d_{\bar{x}}\bar{\eta} \end{pmatrix},$$

and

$$\bar{g} \equiv \frac{1}{4} \begin{pmatrix} 1 & -a^2 & -\bar{\eta}(\bar{x}) \\ a^2 & 2a^2\bar{U}(\bar{x}) & 0 \\ -\bar{\eta}(\bar{x}) & 0 & 0 \end{pmatrix}, \quad (81)$$

$$\bar{V}_A^2(\bar{x}) = a^2\bar{U}(\bar{x}), \quad \bar{V}_A^2(0) = a^2\bar{U}(0) = a^2. \quad (82)$$

For the dimensionless energy flux we have

$$\mathcal{Q}_{\text{wave}}(\bar{x}) = \frac{1}{4} |\hat{u}|^2 + \frac{a^2}{2} \left( \bar{U} |\hat{b}|^2 - \text{Re}(\hat{b}^* \hat{u}) \right) - \frac{1}{2} \bar{\eta} \text{Re}(\hat{u}^* \hat{w}) = \langle \bar{\Psi} | \bar{g} | \bar{\Psi} \rangle, \quad (83)$$

where

$$\hat{u} = \frac{\hat{u}}{U_0}, \quad \hat{w} = \frac{l d_x \hat{u}}{U_0}, \quad \bar{\omega} = \frac{l \omega}{U_0}.$$

Then for dimensional wave energy flux we have

$$q_{\text{wave}}(x) = \rho_0 U_0^3 \mathcal{Q}_{\text{wave}}(\bar{x}), \quad (84)$$

and analogously for the momentum flux of the wave

$$\Pi_{\text{wave}}(x) = \rho_0 U_0^2 \mathcal{P}_{\text{wave}}(\bar{x}), \quad \mathcal{P}_{\text{wave}} = \frac{1}{4} |\hat{b}|^2. \quad (85)$$

in the next section we will consider all parts of the energy and momentum fluxes.

## 2.6. Mass, energy and momentum fluxes

In the one-dimensional model which we now analyze, the conservation laws Eqs. (1)–(3) are converted into three integrals of our dynamic system describing the mass  $j = \rho_0 U_0 \bar{j}$ , energy  $q = \rho_0 U_0^3 \mathcal{Q}$ , and momentum  $\Pi = \rho_0 U_0^2 \mathcal{P}$  fluxes

$$\bar{j} = \bar{U}\bar{\rho} = 1, \quad (86)$$

$$\mathcal{Q} = \frac{1}{2} \bar{U}^2 + c_p \Theta_0 \bar{T} \quad (87)$$

$$\begin{aligned} & -\bar{\kappa} \Theta_0 \bar{F} - \left( \frac{4}{3} \bar{\eta} + \bar{\zeta} \right) \bar{U} \bar{W} \\ & + \frac{1}{4} |\hat{v}|^2 + \frac{a^2}{4} \left( 2\bar{U} |\hat{b}|^2 - \hat{b}^* \hat{v} - \hat{b} \hat{v}^* \right) \\ & - \frac{1}{4} \bar{\eta} \left( \hat{v}^* \hat{w} + \hat{v} \hat{w}^* \right) = \text{const}, \\ & \mathcal{P} = \bar{U} + \frac{\Theta_0 \bar{T}}{\bar{U}} - \left( \frac{4}{3} \bar{\eta} + \bar{\zeta} \right) \bar{W} + \frac{1}{4} |\hat{b}|^2 = \text{const}. \end{aligned} \quad (88)$$

Here we can recognize the energy flux of an ideal inviscid gas

$$\mathcal{Q}_{\text{wind}}^{\text{ideal}} = \frac{1}{2} \overline{U}^2 + c_p \Theta_0 \overline{T}, \quad \Theta = \Theta_0 \overline{T}, \quad (89)$$

dissipative energy flux of the wind related to heat conductivity and viscosity

$$\mathcal{Q}_{\text{wind}}^{\text{diss}} = -\overline{\kappa} \Theta_0 \overline{F} - \left( \frac{4}{3} \overline{\eta} + \overline{\zeta} \right) \overline{U} \overline{W}, \quad (90)$$

the non-absorptive part of wave energy flux

$$\mathcal{Q}_{\text{wave}}^{\text{ideal}} = \frac{1}{4} |\hat{\mathbf{v}}|^2 + \frac{a^2}{2} \left( \overline{U} |\hat{\mathbf{b}}|^2 - \text{Re}(\hat{\mathbf{b}}^* \hat{\mathbf{v}}) \right), \quad (91)$$

and the absorptive part of the wave energy flux

$$\mathcal{Q}_{\text{wave}}^{\text{diss}} = -\frac{1}{2} \overline{\eta} \text{Re}(\hat{\mathbf{v}}^* \hat{\mathbf{w}}) \quad (92)$$

proportional to viscosity. Analogously, for the momentum flux we have:

$$\mathcal{P}_{\text{wind}}^{\text{ideal}} = \overline{U} + \frac{\Theta_0 \overline{T}}{\overline{U}}, \quad (93)$$

$$\mathcal{P}_{\text{wind}}^{\text{diss}} = -\left( \frac{4}{3} \overline{\eta} + \overline{\zeta} \right) \overline{W}, \quad (94)$$

$$\mathcal{P}_{\text{wave}}^{\text{ideal}} = \frac{1}{4} |\hat{\mathbf{b}}|^2, \quad (95)$$

$$\mathcal{P}_{\text{wave}}^{\text{diss}} = 0 \quad (96)$$

for the transversal AWs. As a rule, the dissipative fluxes are against the non-dissipative ones. One can introduce non-ideal wind energy flux

$$\begin{aligned} \tilde{\mathcal{Q}} &\equiv \mathcal{Q}_{\text{wind}}^{\text{nonideal}} = \mathcal{Q}_{\text{wind}}^{\text{diss}} + \mathcal{Q}_{\text{wave}} = \mathcal{Q} - \mathcal{Q}_{\text{wind}}^{\text{ideal}} \\ &\quad - \overline{\kappa} \overline{F} - \left( \frac{4}{3} \overline{\eta} + \overline{\zeta} \right) \overline{U} \overline{W} - \frac{1}{4} \overline{\eta} (\hat{\mathbf{v}}^* \hat{\mathbf{w}} + \hat{\mathbf{v}} \hat{\mathbf{w}}^*) \\ &\quad + \frac{1}{4} |\hat{\mathbf{v}}|^2 + \frac{a^2}{4} \left( 2\overline{U} |\hat{\mathbf{b}}|^2 - \hat{\mathbf{b}}^* \hat{\mathbf{v}} - \hat{\mathbf{b}} \hat{\mathbf{v}}^* \right), \\ \mathcal{Q}_{\text{wave}} &= \mathcal{Q}_{\text{wave}}^{\text{ideal}} + \mathcal{Q}_{\text{wave}}^{\text{diss}}, \end{aligned} \quad (97)$$

and non-ideal wind momentum flux

$$\tilde{\mathcal{P}} \equiv \mathcal{P}_{\text{wind}}^{\text{nonideal}} = \mathcal{P}_{\text{wind}}^{\text{diss}} + \mathcal{P}_{\text{wave}} = \mathcal{P} - \mathcal{P}_{\text{wind}}^{\text{ideal}} = -\left( \frac{4}{3} \overline{\eta} + \overline{\zeta} \right) \overline{W} + \frac{1}{4} |\hat{\mathbf{b}}|^2, \quad \mathcal{P}_{\text{wave}} = \mathcal{P}_{\text{wave}}^{\text{ideal}} + \mathcal{P}_{\text{wave}}^{\text{diss}}. \quad (98)$$

Then according to Eq. (28) we have

$$\sigma^2(\bar{x}) \equiv \tilde{\mathcal{Q}}|_{\bar{x}}^0 = \mathcal{Q}_{\text{wind}}^{\text{ideal}}|_0^{\bar{x}}, \quad (99)$$

$$\tau(\bar{x}) \equiv \tilde{\mathcal{P}}|_{\bar{x}}^0 = \mathcal{P}_{\text{wind}}^{\text{ideal}}|_0^{\bar{x}}. \quad (100)$$

### 3. SHORTWAVELENGTH WKB APPROXIMATION

It is instructive to apply short-wavelength approximation when we have weakly damped plane AWs. The secular equation (21)

$$V_A^2 k^2 = \omega_D (\omega_D + i v_k k^2), \quad \omega_D \equiv \omega - kU, \quad (101)$$

can be solved by successive iterations. In initial approximation  $v_k = 0$  for low frequencies we obtain for the right propagating wave (R-mode)  $k' \approx \omega / (V_A + U)$ . Then in linear with respect to  $v_k = 0$  approximation we derive

$$2k'' \approx \frac{v_k k'^2}{V_A} = \frac{v_k \omega^2}{V_A (V_A + U)^2} \ll k'. \quad (102)$$

Attenuation of the wave amplitude  $\propto e^{-k''x}$  describes the damping of the wave energy density  $\propto e^{-2k''x}$ , or

$$d_x \varepsilon_{\text{wave}} = -2k'' \varepsilon_{\text{wave}}. \quad (103)$$

The wave energy flux obeys the same equation if additionally the dissipative wind energy flux is much smaller than the wave energy flux

$$|\mathcal{Q}_{\text{wind}}^{\text{diss}}| \ll |\mathcal{Q}_{\text{wave}}|, \quad (104)$$

i.e., if heat conductivity and wind viscosity are negligible. Formally this means that in Eq. (87)  $\bar{F} = 0 = \bar{W}$ , i.e.,

$$\left| \frac{d_x T}{T} \right|, \left| \frac{d_x U}{U} \right| \ll k$$

which is the condition for the applicability of the WKB approximation—the medium should be approximately homogeneous. According to Eq. (98) for  $U = \text{const}$  the momentum flux is the time averaged pressure of the AW

$$\tilde{\Pi} \approx \frac{\langle b^2 \rangle_t}{2\mu_0} = \frac{|\hat{b}|^2}{4\mu_0} = \varepsilon_b, \quad (105)$$

which is simultaneously equal to the time averaged magnetic field energy density  $\varepsilon_b$  of the wave. For AW the wave part of the time averaged density of the kinetic energy is equal to the magnetic one

$$\varepsilon_u \approx \frac{1}{2} \rho \langle u^2 \rangle_t = \frac{1}{4} \rho |\hat{u}|^2 = \varepsilon_b. \quad (106)$$

For the averaged energy density of AW we have  $\varepsilon_{\text{wave}} = \varepsilon_u + \varepsilon_b$ . Then if Eq. (97) we neglect all dissipative terms supposing homogeneous medium for a plane AW we obtain

$$\tilde{q}(x) \approx (V_A + U) \varepsilon_{\text{wave}} + U \varepsilon_u. \quad (107)$$

Let us summarize the physical conditions: we suppose runaway waves with small damping, there are no reflected waves by inhomogeneities. This result, notably Eq. (107), should be expected: the wave energy  $\varepsilon_{\text{wave}}$  running with AW velocity  $V_A$  is also drifting with the wind velocity  $U$ . This is an example of the aether drag theory, the energy density is moving with velocity  $V_A + U$ . Additionally, the kinetic energy of the wave,  $\varepsilon_u$ , is blown away by the wind velocity  $U$ .

Contrary to this, for the momentum flux there is no drag, since Maxwell tensor is not affected by the wind  $\tilde{\Pi} \approx \varepsilon_b$ .

A simple integration of Eq. (103) gives  $\tilde{q}(x) = \tilde{q}_0 D(x)$ , where  $k''(x)$  is taken from Eq. (102)

$$D(x) \equiv \exp \left( -2 \int_0^x k''(x) dx \right) = \exp \left\{ - \int_0^x \frac{\omega^2 v_k(x) dx}{[V_A(x) + U(x)]^2 V_A(x)} \right\}. \quad (108)$$

Rewriting Eq. (107) as

$$\tilde{q}(x) = (V_A + U) \varepsilon_{\text{wave}} + U \varepsilon_u = (2V_A + 3U) \varepsilon_b, \quad (109)$$

and using Eq. (105) we obtain the relation

$$\tilde{\Pi} \approx \frac{\tilde{q}(x)}{2V_A(x) + 3U(x)}. \quad (110)$$

For the boundary conditions at  $x = 0$  we suppose that the wind velocity  $U_0$  is much smaller than Alfvén velocity  $V_A(0)$

$$\mathcal{Q}(0) = (2s + 3) \tilde{\mathcal{P}}(0) \gg \tilde{\mathcal{P}}(0), \quad (111)$$

i.e. in this dimensionless units the energy flux of AWs is much greater than momentum flux. Roughly speaking, in the beginning AWs only heat the corona rather than accelerate the wind.

As main qualitative advantage of the MHD calculation of the solar wind problem is the natural explanation of the narrow width of the transition layer  $\lambda$ . Formally we can define

$$\frac{1}{\lambda} = \max \left| \frac{d_x T(x)}{T(x)} \right|. \quad (112)$$

This width can be calculated using the WKB approximation for the energy and momentum fluxes.

#### 4. SELF-CONSISTENT PROCEDURE

First we fix the boundary conditions, namely the temperature  $T_0$  and proton density  $n_p(0)$  for  $x = 0$ . For these parameters we calculate the density  $\rho_0$ , Debye radius  $r_D(0)$ , Coulomb logarithm  $\mathcal{L}_0$ , viscosity  $\eta_0$ , heat conductivity  $\kappa_0$ , Ohmic resistivity  $\rho_{\Omega,0}$ , and sound speed  $c_s(0)$ . The initial velocity of the wind is better to be parametrized by the dimensionless parameter  $s \gg 1$ , i.e.,  $U_0 = c_s(0)/s$ . Analogously, plasma beta parameter  $\beta_0$  determines the Alfvén speed at  $V_A(0) = \sqrt{\gamma/2\beta} c_s(0)$ . Let us also fix the maximal frequency  $\omega$  for which we will consider plasma waves and calculate the absorption rate of the energy density of Alfvén waves  $2k''(0)$ . One can choose the interval of the solution to the MHD equations to be much larger than the AWs mean free path  $1/2k''(0)$ , for example

$$l = \frac{10}{2k''(0)} = \frac{10V_A^3(0)}{v_k(0)\omega^2}. \quad (113)$$

Having units for length  $l$ , velocity  $U_0$  and density  $\rho_0$ , we can calculate dimensionless variables at  $\bar{x} = 0$ :  $\bar{\kappa}_0$ ,  $\bar{\eta}_0$ , and  $\Theta_0$ .

The input parameters of the program are  $T_0$ ,  $n_{\text{tot}}(0)$ ,  $\beta_0$ ,  $\omega$ , and  $s$  which parametrizes  $j = \langle m \rangle n_{\text{tot}}(0)U_0$ . We calculate  $l$ ,  $a$ ,  $\bar{\eta}_0 = \bar{\nu}_0$ ,  $\bar{\kappa}_0$  and choose some  $A_{\text{wave}}$  which finally determines increasing of the temperature  $\bar{T}(1) = T(l)/T(0)$ .

The first step in the self-consistent procedure is to choose some initial approximations for the wind variables, the simplest possible functions are  $\bar{T}(\bar{x}) = \bar{U}(\bar{x}) = 1$  and of course  $\bar{W}(\bar{x}) = \bar{F}(\bar{x}) = 0$ , i.e., to extend the boundary conditions at  $\bar{x} = 0$  for the whole interval  $\bar{x} \in (0, 1)$ . Then one can start the successive approximations.

1. At fixed wind profiles  $\bar{T}(\bar{x})$  and  $\bar{U}(\bar{x})$  we calculate  $\bar{W}$ ,  $\bar{F}$ ,  $\Lambda = \Lambda_0 + \frac{3}{2} \ln \bar{T} + \frac{1}{2} \ln \bar{U}$ ,  $\bar{\eta}$ ,  $\bar{\kappa}$ ,  $d_{\bar{x}} \bar{\eta}$ , and dimensionless matrices  $\bar{\mathbf{M}}$  and  $\bar{\mathbf{g}}$ . Then we have to solve the wave equation, and to renormalize the solution with some fixed dimensionless energy flux for the R-mode  $\mathcal{Q}_{\text{wave}}(0)$ .
2. Using so obtained wave variables  $\Psi$  we have to solve the ordinary differential equations for  $d_{\bar{x}} \bar{F}$  and  $d_{\bar{x}} \bar{W}$ , with boundary conditions, for example,  $\bar{F}(0) = 0$  and  $\bar{W}(0) = 0$ . The variables  $\bar{\eta}$ ,  $\bar{\kappa}$ ,  $d_{\bar{x}} \bar{\eta}$ , and  $d_{\bar{x}} \bar{\kappa}$ , which participate in the coefficients have to be calculated simultaneously.
3. Having solved the equations for the wind variables and formerly the equations for the wave variables we can calculate the total energy and momentum flux. If these fluxes are not constant with the required accuracy we go to item 1 and repeat the procedure.

Now we can calculate the width of the transition layer by the maximal value of the logarithmic derivative of the temperature

$$\frac{1}{\lambda} = \max_{\bar{x}} |d_{\bar{x}} \ln \bar{T}|. \quad (114)$$

The width of the transition layer  $\lambda$ , and the increasing of the temperature  $\bar{T}(1)$  are functions of the wave energy flux coming from the chromosphere  $q_{\text{wave}}(0)$ . We have to repeat the calculations for different  $A_{\text{wave}}$  in order to derive the required temperature enhancement  $\bar{T}(1; A_{\text{wave}}) = T(l)/T(0)$ . At fixed enhancement and chosen spectral density of the AWs we have no more free parameters and have to compare the calculations with another model for heating the solar corona.

## 5. CONCLUSIONS

In spite that iron was suspicious from the very beginning, the problem of Coronium was a 70-year standing mystery until unambiguous identification as  $\text{Fe}^{13+}$  by Grotrian and Edlén in 1939. The same 70-year time quantum was repeated. In 1947 Alfvén [2] advocated the idea that the absorption of AWs is the mechanism of heating the solar corona. Unfortunately the idea by Swedish iconoclast [6] was never realized in original form: what can be calculated, what is measured, what is explained and what is predicted. That is why there is a calamity of ideas still on the arena, for a contemporary review see the SOHO proceedings [7]. From a qualitative point of view the narrow width of the transition layer  $w = \min |dx/d\ln T(x)|$  is the main property which should be compared against the predictions of other scenarios. For example, in order the nanoflare hypothesis to be vindicated [8] such reconnections need to explain the narrow width of the transition layer at the same boundary conditions of wind velocity and temperature. Moreover, electric fields of the reconnections heat mainly the electron component of the plasma. How then proton temperature in the corona is higher? Launching the *Hinode* gave a lot of hints for the existence of AWs in the corona [9], see also [10, 11, 12]. However most of the those research was in the UV region when high-frequency AWs which heated are already absorbed. All observations are for low-frequency (mHz range) AWs for which the hot corona is transparent. The best can be done is to extract the low-frequency behavior of the spectral density of AWs and to extrapolate to higher frequencies responsible for heating. So, observed AWs are irrelevant for heating. In order to identify AWs responsible for heating, it is necessary to investigate the high-frequency (1 Hz range) AWs in the cold chromosphere using optical, but not UV, spectral lines. The first hint in that direction is already found: 20 mHz AWs have been detected [11]. However, we are unaware whether new experiments of such a type are planned. One of the purposes of the present work is to focus the attention of experimentalists on the 1 Hz range AWs in the chromosphere, which we predict on the basis of our MHD analysis. For such purposes we suggest Doppler tomography [13] of  $H_\alpha$  to be used; Ca lines are another possibility. Doppler tomography was successfully used for investigating rotating objects, such as accretion disks [14] and solar tornados [15]. Here we wish to mark also the Doppler tomography by Coronal Multichannel Polarimeter build by Tomczyk [16]. The first observations of periodic variations of full-width-at-half-maximum of  $H_\alpha$  line profile also gave promising results [10]. For investigating AWs by Doppler tomography we suggest the development of frequency dependent Doppler tomography operating as a lock-in voltmeter. The data from every space pixel should be multiplied by  $\sin \omega t$  and integrated for many wave periods. Finally, one can observe the time averaged distribution of the AW amplitude. Systematic investigation of such frequency dependent Doppler tomograms will reveal that Swedish iconoclast [6] is again right that the AWs heat the solar corona, after another 70 years of dramatic cornucopia of ideas.

## ACKNOWLEDGMENTS

One of the authors, T. M., is thankful to Yana Maneva for discussing many problems related to solar physics [17] and Martin Stoev for the early stages of the present research [18] when the idea of self-induced opacity was advocated. He also appreciate fruitful discussions with Eckart Marsch on the problem of the high-frequency Alfvén waves and hospitality of Sami Solanki at the MPI for Solar System Research. This work was partially supported by the St. Clement of Ohrid University at Sofia under Grant 203/2010.

## REFERENCES

1. B. Edlén, *Arkiv f. Matem. Astr. och Fys. B* **28**, 1–4 (1941); B. Edlén, *Zeits. f. Astrophysik* **22**, 30–64 (1942); P. Swings, *Astrophys. J.* **98**, 116–128 (1943).
2. H. Alfvén, <sup>2</sup> *Mon. Not. R. Astron. Soc.* **107**, 211–219 (1947).
3. H. Alfvén, *Nature* **150**, 405–406 (1942).
4. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics: Exercises*, Addison-Wesley, London, 1965, Problems 40.4–5.

---

<sup>2</sup> In this paper entitled “Granulation, magnetohydrodynamic waves and the heating of the solar corona” Alfvén has suggested the heating mechanism of the solar corona. We note that this pionering work regrettably is practically never cited in contemporary reviews on solar corona heating and lounching of the solar wind.

5. L. D. Landau and E. M. Lifshitz, “Mean free path of plasma particles” in *Physical Kinetics*, 2nd edition, Butterworth-Heinemann, Oxford, 2002, sec. 43, Eq. (43.8–10).
6. A. J. Dessler,<sup>3</sup> *Science* **170**, 604–606 (1970);  
<http://www.sciencemag.org/cgi/content/citation/170/3958/604>.
7. *Proceedings of the SOHO 15 Workshop—Coronal Heating* (6–9 September 2004, St. Andrews, Scotland), edited by R. W. Walsh, J. Ireland, D. Danesy, and B. Fleck, ESA SP-575, ESA Publications Division, Noordwijk, 2004.
8. Ch. Day, *Phys. Today* **62**, May 18–21 (2009).
9. B. De Pontieu, S. W. McIntosh, M. Carlsson, V. H. Hansteen, T. D. Tarbell, C. J. Schrijver, A. M. Title, R. A. Shine, S. Tsuneta, Y. Katsukawa, K. Ichimoto, Y. Suematsu, T. Shimizu, and S. Nagata, *Science* **318**, 1574–1577 (2007); T. J. Okamoto, S. Tsuneta, T. E. Berger, K. Ichimoto, Y. Katsukawa, B. W. Lites, S. Nagata, K. Shibata, T. Shimizu, R. A. Shine, Y. Suematsu, T. D. Tarbell, and A. M. Title, *Science* **318**, 1577–1580 (2007); R. Erdélyi and V. Fedun, *Science* **318**, 1572–1574 (2007); Y. Katsukawa, T. E. Berger, K. Ichimoto, B. W. Lites, S. Nagata, T. Shimizu, R. A. Shine, Y. Suematsu, T. D. Tarbell, A. M. Title, and S. Tsuneta, *Science* **318**, 1594–1597 (2007).
10. D. B. Jess, M. Mathioudakis, R. Erdélyi, P. J. Crockett, F. P. Keenan, and D. J. Christian, *Science* **323**, 1582–1585 (2009).
11. J.-S. He, C.-Y. Tu, E. Marsch, L.-J. Guo, S. Yao, and H. Tian, *Astron. Astrophys.* **497**, 525–535 (2009).
12. B. N. Dwivedi and A. K. Srivastava, *Current Sci.* **98**, 295–296 (2010).
13. T. Marsh, *Doppler Tomography*, *Europhysicsnews* **36** No. 4, 133–138 (2005).
14. T. R. Marsh and K. Horn, *Mon. Not. R. Astron. Soc.* **235**, 269–286 (1988).
15. <http://www.mps.mpg.de/en/forschung/sonne/>.
16. S. Tomczyk and S. W. McIntosh, *Asprophys. J.* **697**, 1384–1391 (2009).
17. Y. G. Maneva, “Generation and Dissipation of Alfvén-Cyclotron Turbulence in the Solar Corona and Solar Wind and Related Ion Differential Heating and Acceleration,” Ph.D. Thesis, Göttingen University, 2010.
18. T. M. Mishonov, M. V. Stoev, and Y. G. Maneva, *Eur. Phys. J. D* **44**, 533–536 (2007).

---

<sup>3</sup> Swedish Iconoclast Recognized after Many Years of Rejection and Obscurity